Published by Institute of Physics Publishing for SISSA/ISAS
Received: *April 23, 2002*
Accepted: *July 4, 2002*

Gauging of flat groups in four dimensional supergravity

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Abstract: We show that $N = 8$ spontaneously broken supergravity in four dimensions obtained by Scherk-Schwarz generalized dimensional reduction can be obtained from a pure four dimensional perspective by gauging a suitable electric subgroup of $E_{7,7}$. Owing to the fact that there are non isomorphic choices of maximal electric subgroups of the U-duality group their gaugings give rise to inequivalent theories. This in particular shows that the Scherk-Schwarz gaugings do not fall in previous classifications of possible gauged $N = 8$ supergravities. Gauging of flat groups appear in many examples of string compactifications in presence of brane fluxes.

Keywords: Supersymmetry Breaking, Supergravity Models.

JHEP07(2002)010

Contents

1. Introduction

In recent times, investigation of string theory models whose low energy dynamics is described by a compactification of ten or eleven dimensional supergravity in presence of brane fluxes, has revived the interest in the study of gauged supergravities with different number of supersymmetries, and their possible Higgs and super Higgs phases [1] - [7]. Other examples of models with broken phases are the Scherk-Schwarz [8, 9] supergravities and the so called "no-scale" supergravity models [10, 11]. Models of this type naturally appear when brane fluxes are turned on in orientifold compactifications of type IIB superstring theory, and their no-scale structure gives rise to the interesting possibility of having a dynamically generated hierarchy of scales below the Planck scale [5] - [7]. In the present work we show that all these spontaneously broken models can be viewed as non semisimple gauging of "flat groups" in extended supergravity, originally considered in the generalized dimensional reduction of Scherk-Schwarz [8].

We will mainly consider the example of $N = 8$ gauged supergravity in four dimensions. The existence of a $\mathfrak{so}(1,1)$ grading in the Lie algebra of the duality group $E_{7,7}$ will turn out to be essential. This grading comes from the decomposition

$$\mathfrak{e}_{7,7} = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1) + \mathfrak{p}\,, \qquad \mathfrak{p} = \mathbf{27_{-2}} + \mathbf{27'_{+2}}\,, \tag{1.1}$$

where $\mathfrak{p}$ carries the mentioned representations of $\mathfrak{e}_{6,6} + \mathfrak{so}(1,1)$.

Gauged supergravities based on the above decomposition originate models that differ from previously constructed gaugings [12]–[14] and were overlooked in previous classifications [15]. They allow a construction of the Scherk-Schwarz models from a purely four dimensional perspective.

Although in $N = 8$ supergravity the structure of $E_{7,7}$ is essential we will show that in a large class of models the manifold of the scalars has isometries which include $\mathfrak{so}(1,1)$ and translations. The $\mathfrak{so}(1,1)$ provides a grading of the isometry algebra as well as of the symplectic space of electric and magnetic field strengths. For example, all $N = 2$ supergravities based on special geometry with a cubic prepotential have this property [16, 17]. It is then not surprise that all these models have a five dimensional origin [18].

The flat group whose gauging corresponds to the Scherk-Schwarz mechanism has the following structure,

$$[X_\Lambda, X_0] = f_{\Lambda 0}^{\Sigma} X_\Sigma\,, \tag{1.2}$$

$$[X_\Lambda, X_\Sigma] = 0 \qquad \Lambda = 1, \ldots 27\,, \tag{1.3}$$

where X_Λ is in the $\mathbf{27'_{+2}}$ in (1.1) and X_0 is a generic Cartan generator of $\mathfrak{usp}(8) \subset \mathfrak{e}_{7,7}$. The gauge group is then a semidirect product of two abelian factors. Note that this group is not a subgroup of any previously considered electric subgroup of $\mathrm{E}_{7,7}$. Flat groups of this kind exist in all four dimensional extended supergravity models which have a five dimensional origin, and in particular, in many no-scale supergravity models obtained by turning on brane fluxes. Partial supersymmetry breaking of these models correspond to different Higgs branches of the flat groups.

The paper is organized as follows. In section 2 we discuss non isomorphic electric subgroups of $\mathrm{E}_{7,7}$ and show that a choice which is not a subgroup of $\mathrm{SL}(8,\mathbb{R})$ gives rise to the gauging of the Scherk-Schwarz spontaneously broken supergravity. In section 3 we give the relation between the four dimensional $N = 8$ lagrangian in standard form with the one obtained by Sezgin and van Nieuwenhuizen [19] by generalized dimensional reduction. In particular, we give the relevant terms in the bosonic sector which are due to the gauging of a flat group. Interestingly enough, these terms include unconventional Chern-Simons terms for vector fields, due to the gauging of translational isometries (Peccei-Quinn symmetries) and to the fact that the scalar axions are charged under the gauge group. In section 4 we discuss theories with lower supersymmetries in the perspective of the gauging of flat groups.

2. Electric subgroups of $N = 8$ supergravity

In $N = 8$, $d = 4$ supergravity there are 28 vector gauge potentials Z^Λ_μ. Their field strengths F^Λ, together with their magnetic duals $G_\Lambda = \frac{1}{2}\frac{\partial \mathcal{L}}{\partial\, {}^*F^\Lambda}$[1] transform in the (56 dimensional) fundamental representation of $\mathrm{E}_{7,7}$. $\mathrm{E}_{7,7}$ is embedded in $\mathrm{Sp}(56,\mathbb{R})$ via the fundamental representation, which is then said to be a symplectic representation. This implies that the matrices of the Lie algebra $\mathfrak{e}(7,7)$ are of the form

$$\begin{pmatrix} a & b \\ c & -a^T \end{pmatrix}, \qquad b = b^T\,, \qquad c = c^T\,, \tag{2.1}$$

with a, b, c, d being 28×28 matrices. A generator of the symplectic algebra $\mathfrak{sp}(56)$ has a, b, c arbitrary (b, c symmetric) but the generators belonging to the subalgebra $\mathfrak{e}_{7,7}$ have some restrictions on these entries.

[1] We have used the following normalization for the kinetic lagrangian of the vectors:

$$\mathcal{L} = \mathrm{Im}(\mathcal{N}_{\Lambda\Sigma}) F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \mathrm{Re}(\mathcal{N}_{\Lambda\Sigma}) F^\Lambda_{\mu\nu}\, {}^*F^{\Sigma\mu\nu}\,, \qquad \text{with } {}^*F^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\,.$$

An electric subgroup of $\mathrm{E}_{7,7}$ is any subgroup with $b=0$. Such a subgroup acts linearly on the 28 dimensional space of the electric field strengths (or vector potentials). If the 28 gauge potentials are in the adjoint representation of an electric subgroup, then it is in principle possible to gauge it.

The standard example is SO(8) gauged supergravity [20]. SO(8) is the maximal compact subgroup of $\mathrm{SL}(8,\mathbb{R})$, which is a maximal electric subgroup of $\mathrm{E}_{7,7}$. In the representation (2.1) it has not only $b=0$ but also $c=0$. The reason is that $\mathbf{56}\rightarrow\mathbf{28}+\mathbf{28}'$[2] under $\mathrm{SL}(8,\mathbb{R})$, so the field strengths are not mixed with their duals by an $\mathrm{SL}(8,\mathbb{R})$ transformation.

We would like to give a parametrization of the group $\mathrm{E}_{7,7}$ which depicts the embedding of different electric subgroups. If we think about $N=8$ supergravity in $d=4$ as obtained by dimensional reduction of $N=8$ supergravity in $d=5$, it is pretty obvious that it must be possible to choose an electric subgroup of $\mathrm{E}_{7,7}$ which contains $\mathrm{E}_{6,6}$. Indeed, $\mathrm{E}_{6,6}$ has a linear action on the 27 vector potentials in dimension five, so it will have it also on the dimensionally reduced vectors. The 28$^{\text{th}}$ four dimensional vector comes from the metric, and it is an $\mathrm{E}_{6,6}$ singlet. This suggests that we should look for new electric subgroups of $\mathrm{E}_{7,7}$ by considering the decomposition of the representations of $\mathrm{E}_{7,7}$ under the subgroup $\mathrm{E}_{6,6}\times\mathrm{SO}(1,1)$ (this subgroup is maximal as a reductive subgroup of $\mathrm{E}_{7,7}$, but it is not a maximal subgroup, as we will see later). The fundamental representation decomposes as follows (the subindex indicates the charge under SO(1,1))

$$\mathbf{56}\xrightarrow[\mathrm{E}_{6,6}\times\mathrm{SO}(1,1)]{}\mathbf{27}_{+1}+\mathbf{27}'_{-1}+\mathbf{1}_{+3}+\mathbf{1}_{-3}\,,$$

with $\mathbf{1}_{+3}$ being the new vector that comes from the metric when performing the dimensional reduction. The adjoint representation decomposes as

$$\mathbf{133}\xrightarrow[\mathrm{E}_{6,6}\times\mathrm{SO}(1,1)]{}\mathbf{78}_{0}+\mathbf{1}_{0}+\mathbf{27}_{-2}+\mathbf{27}'_{+2}\,;\tag{2.2}$$

$\mathbf{78_0}$ is the adjoint of $\mathrm{E}_{6,6}$, $\mathbf{1_0}$ is the generator of the rescaling of the radius of S^1, the $\mathbf{27}'_{+2}$ are the shift symmetries acting on the axions coming from the fifth component of the 27 five dimensional vectors, $\mathbf{27}_{-2}$ are the additional transformations, not implementable on the vector potentials, which complete the algebra of $\mathrm{E}_{7,7}$.

From (2.2) it follows that $\mathfrak{e}_{7,7}$ has a grading under $\mathfrak{so}(1,1)$,

$$\mathfrak{e}_{7,7}=\mathfrak{l}_0+\mathfrak{l}_{+2}+\mathfrak{l}_{-2}\,.$$

The representation $\mathbf{56}$ is a graded representation. The underlying vector space is decomposed as $V=V^+\oplus V^-$, being this decomposition compatible with the grading of the Lie algebra. V^+ is the space of the electric field strengths (and potentials) and V^- the space of their magnetic duals. In particular we have that

$$\begin{aligned}X&\in\mathfrak{l}_0\,,\qquad X:V^+\longrightarrow V^+\,,\\X&\in\mathfrak{l}_{+2}\,,\qquad X:V^+\longrightarrow V^+\end{aligned}$$

[2]We will denote by $\mathbf{r}'$ the contragradient representation of $\mathbf{r}$.

so the non semisimple subalgebra $\mathfrak{l}_0+\mathfrak{l}_{+2}$ has a linear action on V^+. Therefore, the matrices of $\mathfrak{l}_0+\mathfrak{l}_{+2}$ in the fundamental representation have $b=0$ (2.1). $\mathfrak{l}_0$ has also $c=0$, while $\mathfrak{l}_{+2}$ has $a\neq 0$, $c\neq 0$. We write the action of $\mathfrak{l}_{\pm 2}$ on $V^+\oplus V^-$:

$$\delta\begin{pmatrix} F^\Lambda \\ F \\ G_\Lambda \\ G \end{pmatrix} = \begin{pmatrix} 0^\Lambda{}_\Sigma & -t'^\Lambda & d^{\Lambda\Sigma\Gamma}t_\Gamma & 0^\Lambda \\ -t_\Sigma & 0 & 0_\Sigma & 0 \\ d_{\Lambda\Sigma\Gamma}t'^\Gamma & 0_\Lambda & 0_\Lambda{}^\Sigma & t_\Lambda \\ 0_\Sigma & 0 & t'^\Sigma & 0 \end{pmatrix} \begin{pmatrix} F^\Sigma \\ F \\ G_\Sigma \\ G \end{pmatrix}. \tag{2.3}$$

t_Λ and t'^Λ are the parameters of the transformation and $d_{\Lambda\Sigma\Gamma}$ is the symmetric invariant tensor of the representation **27** of $\mathrm{E}_{6,6}$ ($d^{\Lambda\Sigma\Gamma}$ of the $\mathbf{27}'$). The matrices of $\mathfrak{l}_{-2}$ have $c=0$, $a\neq 0, b\neq 0$. We denote the vector potentials as $(Z^\Lambda_\mu, Z^0_\mu = B_\mu,\ \Lambda=1,\dots 27)$. They transform as V^+, so we have

$$\begin{aligned} \delta_{\mathbf{27}'_{+2}} Z^\Lambda_\mu &= t'^\Lambda B_\mu \qquad & \delta_{\mathbf{1}_0} Z^\Lambda_\mu &= \lambda Z^\Lambda_\mu \\ \delta_{\mathbf{27}'_{+2}} B_\mu &= 0 & \delta_{\mathbf{1}_0} B_\mu &= 3\lambda B_\mu\,. \end{aligned}$$

The above transformation properties under translations and $\mathrm{SO}(1,1)$ are common to a large class of supergravity models with different number of supersymmetries.

Finally, let us discuss the scalar sector. The coset of the scalars in four and five dimensions are respectively $\mathrm{E}_{7,7}\,/\,\mathrm{SU}(8)$ and $\mathrm{E}_{6,6}\,/\,\mathrm{USp}(8)$. The corresponding Cartan decompositions are

$$\begin{aligned} \mathfrak{e}_{7,7} &= \mathfrak{su}(8)+\mathfrak{p}\,, \qquad & \mathfrak{p} &= \mathbf{70} \text{ of } \mathrm{SU}(8) \\ \mathfrak{e}_{6,6} &= \mathfrak{usp}(8)+\mathfrak{p}' & \mathfrak{p} &= \mathbf{42} \text{ of } \mathrm{USp}(8)\,, \end{aligned}$$

and we have that the **70** of SU(8) is decomposed

$$\mathbf{70} \xrightarrow[\mathrm{USp}(8)]{} \mathbf{42}+\mathbf{27}+\mathbf{1}\,.$$

The physical meaning of the above decomposition is that the scalars in the **27** come from the fifth component of the 27 five dimensional vectors and the singlet from the g_{55} component of the metric (radius of S^1).

3. Standard form of $N=8$ Scherk-Schwarz supergravity and gauging of flat groups

We want to compare the maximally extended ($N=8$) supergravity in four dimensions with the theory found by Sezgin and Van Nieuwenhuizen [19] through the Scherk-Schwarz dimensional reduction from five dimensions.

Let us first consider the case where all the mass parameters are set to zero (standard dimensional reduction). In dimension five the U-duality group is $\mathrm{E}_{6,6}$, and it acts linearly on the 27 vector potentials $\hat{A}^\Lambda_{\hat\mu}$, with $\hat\mu=1,\dots,5$ and $\Lambda=1,\dots,27$. We will denote the quantities in five dimensions with a hat, " ˆ ", to distinguish them from the four dimensional ones. The local symmetries acting on these vector potentials are general

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coordinate transformations in five dimensions with parameters $\hat{\xi}^{\hat{\mu}}(x^{\hat{\nu}})$ and 27 abelian U(1) gauge transformations $\Xi^\Lambda(x^{\hat{\nu}})$. When performing the reduction to four dimensions, the local symmetries that remain are: four dimensional general coordinate transformations with parameters $\xi^\mu(x^\nu)$ with $\mu, \nu = 1, \ldots, 4$, a gauge transformation with parameter $\xi^5(x^\nu)$, and the U(1) gauge transformations with parameters $\Xi^\Lambda(x^\nu)$. Explicitly these transformations read

$$\begin{aligned}
\delta_{\xi^5} A^\Lambda_\mu &= \partial_\mu \xi^5 A^\Lambda_5 \\
\delta A^\Lambda_\mu &= \partial_\mu \Xi^\Lambda \\
\delta_{\xi^5} A^\Lambda_5 &= 0 \\
\delta_\Xi A^\Lambda_5 &= 0\,,
\end{aligned}$$

We will denote $A^\Lambda_5 = a^\Lambda$. There is also a global (x-independent) invariance $a^\Lambda \to a^\Lambda + t'^\Lambda$. We denote by B_μ and ϕ the vector and scalar coming from the reduction of the vielbein,

$$\hat{V}^{\hat{a}}_{\hat{\mu}} = (V^a_\mu, V^5_\mu = V^5_5 B_\mu, V^5_5 = e^{2\phi})$$

with transformation

$$\delta_{\xi^5} B_\mu = \partial_\mu \xi^5\,.$$

The combinations

$$Z^\Lambda_\mu = A^\Lambda_\mu - a^\Lambda B_\mu$$

are inert under ξ^5 and so Z^Λ_μ and B_μ are genuine four dimensional gauge fields. Note, however, that under the global translation t'^Λ, Z^Λ transforms

$$\delta Z^\Lambda_\mu = -t'^\Lambda B_\mu\,.$$

The meaning of this transformation is that the 28 vectors (Z^Λ_μ, B_μ) form a 28 dimensional indecomposable representation of the 27 dimensional translation group (t'^Λ). The action given in ref. [19] is also invariant under the following SO(1,1) transformation (with parameter λ)

$$\begin{aligned}
\phi' &= \phi - \lambda \\
Z'^\Lambda_\mu &= e^\lambda Z^\Lambda_\mu \\
B'_\mu &= e^{3\lambda} B_\mu \\
a'^\Lambda &= e^{-2\lambda} a^\Lambda\,.
\end{aligned}$$

We observe that the group generated by SO(1,1) and the 27 global translations is precisely the same as the one discussed in section 2, which appears in the decomposition of $E_{7,7}$ under $E_{6,6}$.

In terms of the fields $\phi, a^\Lambda, B_\mu, Z^\Lambda_\mu$, the lagrangian (4.33) of [19] at zero masses reduces to the following standard expression:[3]

$$\begin{aligned}\mathcal{L}^{\rm bos}_{d=4} &= -\frac{1}{4}VR + \frac{3}{2}V\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}Ve^{-4\phi}\hat{\mathcal{N}}_{\Lambda\Sigma}\partial_\mu a^\Lambda\partial^\mu a^\Sigma + \frac{1}{24}VP^{abcd}_\mu P^\mu_{abcd} + \\ &\quad + V{\rm Im}(\mathcal{N}_{00})B_{\mu\nu}B^{\mu\nu} + 2V{\rm Im}(\mathcal{N}_{0\Lambda})Z^\Lambda_{\mu\nu}B^{\mu\nu} + V{\rm Im}(\mathcal{N}_{\Lambda\Sigma})Z^\Lambda_{\mu\nu}Z^{\Sigma\mu\nu} + \\ &\quad + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\left[{\rm Re}(\mathcal{N}_{00})B_{\mu\nu}B_{\rho\sigma} + 2{\rm Re}(N_{\Lambda 0})B_{\mu\nu}Z^\Lambda_{\rho\sigma} + {\rm Re}(\mathcal{N}_{\Lambda\Sigma})Z^\Lambda_{\mu\nu}Z^\Sigma_{\rho\sigma}\right] \end{aligned} \tag{3.1}$$

in terms of the 28×28 complex symmetric matrix

$$\begin{aligned}\mathcal{N}_{00} &= \frac{1}{3}d_{\Lambda\Sigma\Gamma}a^\Lambda a^\Sigma a^\Gamma - \frac{\rm i}{2}\left(e^{2\phi}a^\Lambda a^\Sigma\hat{\mathcal{N}}_{\Lambda\Sigma} + \frac{1}{2}e^{6\phi}\right)\\ \mathcal{N}_{\Lambda 0} &= \frac{1}{2}d_{\Lambda\Sigma\Gamma}a^\Sigma a^\Gamma - \frac{\rm i}{2}e^{2\phi}\hat{\mathcal{N}}_{\Lambda\Sigma}a^\Sigma\\ \mathcal{N}_{\Lambda\Sigma} &= d_{\Lambda\Sigma\Gamma}a^\Gamma - \frac{\rm i}{2}e^{2\phi}\hat{\mathcal{N}}_{\Lambda\Sigma}\,.\end{aligned}$$

P^{abcd}_μ is the $E_{6,6}\,/\,{\rm USp}(8)$ vielbein, $V = \det V^a_\mu$ and $\hat{\mathcal{N}}_{\Lambda\Sigma}$ is the five dimensional (SO(1,1) invariant) vector kinetic matrix.

The representation of $E_{7,7}$ on the 56 dimensional vector space of the field strengths and their duals is symplectic. For the moment being, let $\mathcal{F}$ denote the vector (F^Λ, F) and $\mathcal{G}$ (G_Λ, G). In terms of the self dual combinations $\mathcal{F}^+ = \frac{1}{2}(\mathcal{F} + {\rm i}\,{}^*\mathcal{F})$, $\mathcal{G}^+ = \frac{1}{2}(\mathcal{G} + {\rm i}\,{}^*\mathcal{G})$, a generic element of the symplectic group would act as

$$\begin{pmatrix}\mathcal{F}'^+\\ \mathcal{G}'^+\end{pmatrix} = \begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}\mathcal{F}^+\\ \mathcal{G}^+\end{pmatrix}, \qquad A^TC \text{ and } B^TD \text{ symmetric}, \qquad A^TD - C^TB = \mathbb{1}\,.$$

Then the matrix $\mathcal{N}$ transforms as

$$\mathcal{N}' = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}\,.$$

The fractional formula allows us to compute the non linear transformations of the fields under the upper block triangular transformation corresponding to the $\mathbf{27_{-2}}$ elements of $\mathfrak{e}_{7,7}$ in (2.3),

$$\begin{aligned}\delta\mathcal{N}_{\Lambda\Sigma} &= -\mathcal{N}_{\Lambda\Pi}\mathcal{N}_{\Delta\Sigma}d^{\Pi\Delta\Gamma}t_\Gamma + t_\Lambda\mathcal{N}_{\Sigma 0} + t_\Sigma\mathcal{N}_{\Lambda 0}\\ \delta\mathcal{N}_{\Lambda 0} &= -\mathcal{N}_{\Lambda\Sigma}\mathcal{N}_{\Delta 0}d^{\Sigma\Delta\Gamma}t_\Gamma + t_\Lambda\mathcal{N}_{00}\\ \delta\mathcal{N}_{00} &= -\mathcal{N}_{\Lambda 0}\mathcal{N}_{\Gamma 0}d^{\Lambda\Gamma\Delta}t_\Delta\end{aligned}$$

The infinitesimal transformation of $\mathcal{N}$ under the translational symmetries $\mathbf{27'_{+2}}$ (lower block triangular) in (2.3) is

$$\begin{aligned}\delta\mathcal{N}_{\Lambda\Sigma} &= d_{\Lambda\Sigma\Delta}t'^\Delta\\ \delta\mathcal{N}_{\Lambda 0} &= \mathcal{N}_{\Lambda\Sigma}t'^\Sigma\\ \delta\mathcal{N}_{00} &= 2\mathcal{N}_{\Lambda 0}t'^\Lambda\end{aligned}$$

[3] Note that our definitions slightly differ from the ones in [19]. In particular, we have defined the field strengths of the vectors as: $Z^\Lambda_{\mu\nu} = \frac{1}{2}(\partial_\mu Z^\Lambda_\nu - \partial_\nu Z^\Lambda_\mu)$, $B_{\mu\nu} = \frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)$. Moreover, with respect to [19] we have redefined $\phi/\sqrt{3} \to \phi$ and $2d_{\Lambda\Sigma\Delta} \to d_{\Lambda\Sigma\Delta}$.

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In particular, we note that only $\mathrm{Re}(\mathcal{N}_{\Lambda\Sigma})$ (theta term) transforms non linearly under the translations t'.

We consider now the semidirect product of the 27 translations with parameters t'^{Λ} with a generic element of the Cartan subalgebra of USp(8), maximal compact subgroup of $E_{6,6}$. Its Lie algebra is of the form (1.3) with $f_{\Lambda 0}^{\Sigma}$ the matrix of the Cartan element in the representation **27** of USp(8). We will denote it by M_{Λ}^{Σ}. Since USp(8) has rank four, M_{Λ}^{Σ} depends on four parameters, m_i, $i = 1, \ldots 4$ and its 27 eigenvalues are functions of these four paramenters. They are given by [8]

$$\pm\mathrm{i}(m_i \pm m_j)\,, \qquad i<j$$

and 3 eigenvalues are 0.[4] This means that there are 3 linear combinations of X_Λ which actually commute with X_0.

We choose this group to perform the gauging of $N = 8$ supergravity. Z_μ^Λ are the gauge connections for X_Λ generators and B_μ is the U(1) gauge connection. It is straightforward to see that the gauge transformations of the connection fields are as follows

$$\begin{aligned}
\delta Z_\mu^\Lambda &= \partial_\mu \Xi^\Lambda + \Xi^0 M_\Sigma^\Lambda Z_\mu^\Sigma - \Xi^\Sigma M_\Sigma^\Lambda B_\mu \\
\delta B_\mu &= \partial_\mu \Xi^0\,.
\end{aligned}$$

The field strengths are

$$\begin{aligned}
F_{\mu\nu}^\Lambda &= \frac{1}{2}\left(\partial_\mu Z_\nu^\Lambda - \partial_\nu Z_\mu^\Lambda - M_\Sigma^\Lambda (Z_\nu^\Sigma B_\mu - Z_\mu^\Sigma B_\nu)\right) \\
B_{\mu\nu} &= \frac{1}{2}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right).
\end{aligned}$$

The gauge transformation of the axion fields a^Λ is

$$\delta a^\Lambda = M_\Sigma^\Lambda \Xi^\Sigma + \Xi^0 M_\Sigma^\Lambda a^\Sigma\,,$$

and their covariant derivatives

$$\nabla_\mu a^\Lambda = \partial_\mu a^\Lambda - M_\Sigma^\Lambda a^\Sigma B_\mu - M_\Sigma^\Lambda Z_\mu^\Sigma\,.$$

In order to write the gauge completion of the lagrangian (3.1), we first observe that $\hat{\mathcal{N}}_{\Lambda\Sigma}$ is invariant under the gauge transformations Ξ^Λ, but transforms under Ξ^0 as follows

$$\delta\hat{\mathcal{N}}_{\Lambda\Sigma} = \Xi^0 M_\Lambda^\Delta \hat{\mathcal{N}}_{\Sigma\Delta} + (\Lambda \leftrightarrow \Sigma)\,.$$

It then follows that under Ξ^0

$$\begin{aligned}
\delta\mathcal{N}_{\Lambda\Sigma} &= \Xi^0 M_\Lambda^\Delta \mathcal{N}_{\Sigma\Delta} + (\Lambda \leftrightarrow \Sigma) \\
\delta\mathcal{N}_{\Lambda 0} &= \Xi^0 M_\Lambda^\Delta \mathcal{N}_{\Sigma 0} \\
\delta\mathcal{N}_{00} &= 0\,,
\end{aligned}$$

[4]The physical masses of the vector bosons actually are proportional to the above eigenvalues with a moduli dependent prefactor $e^{-3\phi}$, as it can be seen from inspection of the lagrangian (3.1). The same observation holds also for the masses of the other particles, as given in [8, 9].

and under Ξ^Λ

$$\begin{aligned}
\delta\mathcal{N}_{\Lambda\Sigma} &= d_{\Lambda\Sigma\Delta}M^{\Delta}_{\Pi}\Xi^{\Pi}\\
\delta\mathcal{N}_{\Lambda 0} &= \mathcal{N}_{\Lambda\Sigma}M^{\Sigma}_{\Pi}\Xi^{\Pi}\\
\delta\mathcal{N}_{00} &= 2\mathcal{N}_{\Lambda 0}M^{\Lambda}_{\Pi}\Xi^{\Pi}\,.
\end{aligned}$$

The gauge completion of the lagrangian (3.1) is then obtained by replacing

$$\begin{aligned}
F^{\Lambda}_{\mu\nu} &\mapsto F^{\Lambda}_{\mu\nu} \text{ non abelian}\\
\partial_\mu a^\Lambda &\mapsto \nabla_\mu a^\Lambda\\
P^{abcd}_{\mu} &\mapsto P^{abcd}_{\mu} - P^{abcd}_{5}B_\mu\,,
\end{aligned}$$

where P^{abcd}_5 is defined in ref. [19, (4.3) and (4.33)] and has the property that

$$\delta_{\Xi^0}P^{abcd}_{\mu} = P^{abcd}_{5}\partial_\mu\Xi^0,$$

and adding the extra term

$$\mathcal{L}^{\text{extra}} = \frac{1}{3}d_{\Lambda\Sigma\Pi}M^{\Pi}_{\Delta}\epsilon^{\mu\nu\rho\sigma}Z^{\Lambda}_{\mu}Z^{\Delta}_{\nu}\left(\partial_\rho Z^{\Sigma}_{\sigma} - \frac{3}{4}M^{\Sigma}_{\Gamma}Z^{\Gamma}_{\rho}B_\sigma\right). \tag{3.2}$$

Note that the second term in (3.2) is in fact identically zero here, due to the symmetry property of $d_{\Lambda\Sigma\Pi}$.

In the Scherk-Schwarz theory, the term (3.2) comes from the generalized dimensional reduction of the Chern-Simons term [19], but in a four dimensional setting it is required because of the non linear transformation

$$\delta\,\mathrm{Re}(\mathcal{N}_{\Lambda\Sigma}) = d_{\Lambda\Sigma\Pi}M^{\Pi}_{\Gamma}\Xi^{\Gamma}\,,$$

as shown in ref. [16, (3.16)].

Here $d_{\Lambda\Sigma\Pi}M^{\Pi}_{\Gamma} = C_{\Gamma,\Lambda\Sigma}$ of [16] and it satisfies the property

$$C_{\Gamma,\Lambda\Sigma} + C_{\Lambda,\Sigma\Gamma} + C_{\Sigma,\Gamma\Lambda} = 0$$

as a consequence of the fact that $d_{\Lambda\Sigma\Pi}$ is an invariant tensor of $E_{6(6)}$.

Supersymmetry also requires a scalar potential

$$U = \frac{1}{24}e^{-6\phi}P_{5abcd}P^{abcd}_{5}$$

as shown in ref. [19]. We do not discuss here the fermionic sector of the theory whose gauge completion can also be obtained in a standard manner.

4. Gauging of flat groups in $N = 2$ four dimensional supergravities

We want to extend the gauging of flat groups to a class of $N = 2$ four dimensional supergravities which have a five dimensional interpretation. The graviton multiplet has fields

$$\left(V^{a}_{\mu}, \psi_{\mu A}, Z^{0}_{\mu}\right).$$

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It is well known that the interactions of $N=2$ vector multiplets with fields

$$\left(Z^\Lambda_\mu, \lambda^\Lambda_A, z^\Lambda\right), \qquad \Lambda = 1, \dots n_v \,,$$

are described by the special geometry of the scalar sigma model. The special geometry [16] of the Kaehler manifold of the scalar fields is completely specified by an holomorphic prepotential of n_v+1 variables, homogeneous of degree two

$$\mathcal{F}\left(x^\Lambda, x^0\right) = {x^0}^2 f\left(\frac{x^\Lambda}{x^0}\right),$$

where in special coordinates,

$$z^\Lambda = \frac{x^\Lambda}{x^0}\,.$$

The holomorphic functions

$$\left(x^\Lambda, x^0, F_\Lambda = \frac{\partial \mathcal{F}}{\partial x^\Lambda}, F_0 = \frac{\partial \mathcal{F}}{\partial x^0}\right)$$

are a local section of a flat symplectic bundle (with structure group $\mathrm{Sp}(2n_v+2,\mathbb{R})$) over the special Kaehler manifold [21, 22]. The duality group G is a subgroup of $\mathrm{Sp}(2n_v+2,\mathbb{R})$. The representation of G acting on the field strengths $(F^{+\Lambda}_{\mu\nu}, F^{+0}{}_{\mu\nu})$ and their duals $(G^+{}_{\Lambda\mu\nu}, G^+{}_{0\mu\nu})$ is a symplectic representation.

If we choose a cubic prepotential of the form

$$\mathcal{F} = \frac{1}{3!}\frac{c_{\Lambda\Sigma\Delta}x^\Lambda x^\Sigma x^\Delta}{x^0}\,,$$

then the Kaehler manifold has always n_v+1 isometries which form a group $\mathrm{SO}(1,1)\circledS\mathbb{R}^{n_v}$ (semidirect product) acting on the coordinates x as follows

$$\begin{aligned}
\delta x^\Lambda &= \lambda x^\Lambda, & \delta x^0 &= 3\lambda x^0 \\
\delta x^\Lambda &= {t'}^\Lambda x^0 & \delta x^0 &= 0\,.
\end{aligned}$$

This transformation induces a symplectic action on the symplectic sections and the electromagnetic field strengths as follows

$$\begin{pmatrix}
\lambda & {t'}^\Lambda & 0 & 0 \\
0 & 3\lambda & 0 & 0 \\
c_{\Lambda\Sigma\Delta}{t'}^\Delta & 0 & -\lambda & 0 \\
0 & 0 & -{t'}^\Lambda & -3\lambda
\end{pmatrix}.$$

The lower block triangular transformations have the same form of the $\mathfrak{l}_0+\mathfrak{l}_+$ of $\mathfrak{e}_{7,7}$. This should not come as a surprise since all these theories can be obtained by dimensional reduction of $N=2$ supergravity in five dimensions with n_v-1 vector multiplets whose real special geometry is defined by the cubic surface [18]

$$c_{\Lambda\Sigma\Delta}y^\Lambda y^\Sigma y^\Delta = 1\,.$$

G'/H'	G/H	$\mathfrak{r}(n_v)$	$\mathfrak{r}(2n_v+2)$	$\dim(\mathrm{CSA})_{H'}$
$\mathrm{SL}(3,\mathbb{R})/\,\mathrm{SO}(3)$	$\mathrm{Sp}(6,\mathbb{R})/\,\mathrm{U}(3)$	**6**	**14**	1
$\mathrm{SL}(3,\mathbb{C})/\,\mathrm{SU}(3)$	$\mathrm{U}(3,3)/\,\mathrm{U}(3)\times\mathrm{U}(3)$	**9**	**20**	2
$\mathrm{SU}^*(6)/\,\mathrm{USp}(6)$	$\mathrm{SO}^*(12)/\,\mathrm{U}(6)$	**15**	**32**	3
$\mathrm{E}_{6,-26}\,/\,\mathrm{F}_4$	$\mathrm{E}_{7,-25}\,/\,\mathrm{E}_6\times\mathrm{SO}(2)$	**27**	**56**	4

Table 1: Exceptional $N=2$ supergravities.

If the real special geometry in five dimensions has no isometries ($c_{\Lambda\Sigma\Delta}$ are generic), and in the absence of hypermultiplets, the only global symmetry in five dimensions is the SU(2) R-symmetry and the Scherk-Schwarz generalized dimensional reduction would correspond to the gauging of the U(1) Cartan element of SU(2), giving mass only to the gravitinos and the gauginos, which are the only fields charged under this U(1). Supersymmetry would be broken while the $\mathrm{U}(1)\,\circledS\,\mathbb{R}^{n_v}$ group would remain unbroken.

In the case where there is a group of isometries G' acting on the real special manifold (for dimension 5), then we could use for the Scherk-Schwarz mechanism a U(1) which has also a component on the Cartan element of the maximal compact subgroup of G'. A non trivial flat group would emerge in four dimensions whose gauging would be similar to the gauging of $N=8$ supergravity discussed in section 2 .

As an illustration, let us consider models where the real special geometry is a coset space G'/H' and the n_v five dimensional vectors belong to a linear representation $R(n_v)$ of G'. Kaluza-Klein dimensional reduction implies that the U-duality group in four dimensions G has an SO(1,1) grading when decomposing with respect to $G'\subset G$

$$\mathfrak{g}=\mathfrak{g}'_0+\mathbf{1_0}+\mathfrak{r}(n_v)_{-\mathbf{2}}+\mathfrak{r}'(n_v)_{+\mathbf{2}}\,.$$

($\mathfrak{r}$ denotes the representation space of R_{n_v}). Moreover, the $2n_v+2$ dimensional symplectic representation $R(2n_v+2)$ of the four dimensional duality group G has the following decomposition under $G'\times\mathrm{SO}(1,1)$

$$\mathfrak{r}(2n_v+2)=\mathfrak{r}(n_v)_{+\mathbf{1}}+\mathbf{1}_{+\mathbf{3}}+\mathfrak{r}'(n_v)_{-\mathbf{1}}+\mathbf{1}_{-\mathbf{3}}\,.$$

Note that these decompositions and the SO(1,1) grading are universal and do not depend on the choice of G'. Indeed, this grading has a Kaluza-Klein origin.

We give in table 1 the examples corresponding to the exceptional five dimensional cosets. The flat group of dimension n_v+1 has structure constants which depend on $\dim(\mathrm{CSA})_{H'}+1$ parameters.

Most of the spontaneously broken models studied in ref. [23] have a dynamical origin analogous to the one discussed in the present investigation. However, since there are models which break an odd number of supersymmetries, one can consider gaugings that do not have a five dimensional interpretation. This should be the case for some models obtained by turning on brane fluxes [5, 6, 7].

JHEP07(2002)010

Acknowledgments

S. F. would like to thank Raymond Stora for enlightening conversations and the Dipartimento di Fisica, Politecnico di Torino for its kind hospitality during the completion of this work. Work supported in part by the European Comunity's Human Potential Program under contract HPRN-CT-2000-00131 Quantum Space-Time, in which L. A., R. D. and M.A. Ll. are associated to Torino University. The work of S. F. has also been supported by the D.O.E. grant DE-FG03-91ER40662, Task C.

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